6

Answers and Hints

6.13 Reputational Equilibrium

If it is worthwhile for the firm to lie when it claims its good has quality $q > 0$, it might as well set its actual quality to 0, since the firm minimizes costs this way. Its profits are then

$$\pi_f = (4 + 6q_a - x - 2)x = (2 + 6q_a - x)x.$$

Profits are maximized when

$$\frac{d\pi_f}{dx} = 2 + 6q_a - 2x = 0,$$

so $x = 1 + 3q_a$, and $\pi_f = (1 + 3q_a)^2$.

Now suppose the firm tells the truth. Then its per-period profits are

$$\pi_t = (2 + 6q_a - 6q_a^2 - x)x,$$

and

$$\frac{d\pi_t}{dx} = 2 + 6q_a - 6q_a^2 - 2x = 0,$$

so $x = 1 + 3q_a - 3q_a^2$, and $\pi_t = (1 + 3q_a - 3q_a^2)^2$.

But total profits Π from truth-telling is π_t forever, discounted at rate $\delta = 0.9$, or

$$\Pi = \frac{\pi_t}{1 - \delta} = 10(1 + 3q_a - 3q_a^2)^2.$$

Truth-telling is profitable then when $\Pi \geq \pi_f$, or when

$$10(1 + 3q_a - 3q_a^2)^2 > (1 + 3q_a)^2. \tag{6.1}$$

Note that equation 6.1 is true for very small q_a (i.e., q_a near 0), and false for very large q_a (i.e., q_a near 1).

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